FORENSIC SCIENCE JOURNAL SINCE 2002

Forensic Science Journal
2003;2:35-46

Preliminary study on speed evaluation and impact configuration reconstruction of vehicles with human-machine interactive programming

M.-Ch. Pan,[1,*]; Y.-S. Chiang,[2]; M.-H. Huang,[3]

[1] *Associate Professor, Department of Mechanical Engineering, National Central University, Jung-Li 320, TAIWAN, ROC*

[2] *Professor, Department of Forensic Science, Central Police University, Kwei-Shan 333, TAIWAN, ROC*

[3] *Graduate Student, The Graduate Institute of Mechanical Engineering, National Central University, Jung-Li 320, TAIWAN, ROC*

Received: October 23, 2002/ Received in revised from: February 19, 2003/ Accepted: February 20, 2003

Abstract

The study aims at developing analysis approaches of vehicle-accident reconstruction in an attempt to recreate the initial impact configuration and clarify accident responsibility. One purpose is to conclude a straightforward and confident reconstruction module as a base of dealing with vehicle impact accidents. Both forward and backward deriving methods have been applied as the means of accident reconstruction. The initial impact configuration can be resulted from dynamic equations of pre-impact, impact and post-impact phases. The obtained formulated relations have been further schemed to be an interactive human-machine environment. The initial impact configuration can thus be yielded, as some necessary data are input to the developed program.

Key Words: Accident Reconstruction, Vehicle Dynamics, Forensic Engineering, Man-Machine Interactive Programming.

1. Introduction

Traffic accident reconstruction relating studies have drawn lots of attention for years in the field of automobile and transportation engineering. Especially in the USA, several factors, such as vehicle-crash simulation and test techniques developed in individual automobile manufacturers, intensive domestic highway network systems, and the necessity of precise impact configuration due to daily heavy traffic accidents etc, make the research topics nourished. In Taiwan, these investigating topics are neglected as industrial investment and development has focused on electronic products for past decades, which resulted in insufficient research capabilities of automobile engineering. Recently, rapid increase of motor vehicles has caused traffic accidents more frequent and serious than ever. The situation motivates the necessity of conducting studies regarding accident reconstruction beside of only introducing commercialized computer programs to this purpose.

Rreconstruction of traffic accidents can both recreate the initial impact configuration and help clarify responsibility. Velocity (i.e., driving speed and its direction) estimation for two (or more) vehicles before accidents is the crucial component of accident reconstruction. Some major factors relating to vehicle velocity include vehicle weight, moment inertia of vehicle body, distortion severity of vehicle, braking traces, and restitution coefficient after collision. Besides, the collision model also takes into account of final position after collision, friction coefficient of tire-road interface, and road profile etc. Brach's monograph (1991) conducted dynamic-equation derivation for two-vehicle collision. Ishikawa (1993) especially investigated the effect of normal and tangential reinstitution coefficients on impact model. The simulation models for the analy-

* Corresponding author, e-mail: pan_minc@cc.ncu.edu.tw

sis of traffic accidents can basically be classified in two types based on the basic laws of physics: dynamic models (Ishikawa, 1985), and analytic models (McHenry, 1997). Having developed analytic collision models, among the most well known are CRASH (Day and Hargens, 1987), and OLDMISS (Prasad, 1991). Another way to study traffic accidents is to apply the finite element methods and structural analysis models for investigating the behavior of the vehicle's structure at the moment of collision. The most impressive programs of this approach applied to crash analysis are PAMCRASH and DYNA3D (Wirley and Engelmann, 1993). As to the state-of-the-art of accident-reconstruction researches in Taiwan, Wang (1999) put an effort on identifying tire marks for clarifying accident responsibility. In addition, although some research programs supported by the National Science Council were conducted in the past ten years, no published studies can be found.

To properly cope with accident-reconstruction tasks, developing the analysis techniques of two-vehicle impact model is significant. The research aims at developing analysis approaches of vehicle-accident reconstruction in an attempt to recreate the initial impact configuration. Especially, an interactive human-machine interface scheme is being developed and constructed on the LabVIEW programming language. Compare the work with previous ones, analyzers can simply insert inputs on the interactive program in the LabVIEW environment, and then the interesting acciednt-reconstruction parameters, which have been derived from the direct- and inverse-derivation equations, can be obtained promptly.

2. Theoretical Basis

The sequence of vehicle collision can be divided in three phases, i.e., pre-collision, collision and post- collision etc. The section will estimate vehicle velocity before collision via deriving kinetic equations of each phase.

2-1 Pre- and Post-collision Phase

All derivation assumes vehicles in rigid body motion during each collision phase. Action forces on a vehicle consider both impact force from the other vehicle, and braking force between tires and road surface. Additionally, a sliding mode is applied during the pre-collision phase in a condition of braking lock. During the post-collision phase, a sliding associated with rotating mode is based to derive relative dynamic equations.

2-1-1 Sliding during Pre-collision Phase

During the pre-collision phase, vehicle speed, V_t, decreases due to sliding friction force applied on tires (Hibbeler, 1986).

$$V_t = V - a_1 t \,, \tag{1}$$

where the deceleration (a_1) obtained from multiplication of gravity (g) and tire-road friction coefficient (m_{tr}), i.e. a_1=gm_{tr} ; V denotes vehicle speed at the instant of braking applied. The braking distance (S_{pre}) relates to vehicle speed as

$$V_t^{\,2} = V^{\,2} - 2a_1 S_{pre} \tag{2}$$

2-1-2 Sliding during Post-collision Phase

After collision, vehicle speed (U_t) that decelerates along sliding can be expressed as (Hibbeler, 1986)

$$U_t = U - a_2 t \,, \tag{3}$$

where , similar to the pre-collision phase; U denotes initial vehicle speed after collision. Meanwhile, sliding distance (S_{post}) relates vehicle speed as

$$U_t^{\,2} = U^2 - 2a_2 S_{post} \,. \tag{4}$$

In addition to sliding, a rotating motion around mass center can be yielded resulting from eccentric collision, i.e., the collision line not crossing mass center. The rotating angular velocity (ω_t) decelerates due to sliding friction force on tires, i.e.,

$$\omega_t = \omega - \alpha t \tag{5}$$

where denotes initial angular velocity after collision, and $\alpha = T/I$ is the angular deceleration which results from friction torque (T) exerted on four wheels of the vehicle around the mass center. Let S_x and S_y be the spans of left/right and front/rear wheels, respectively, and the mass center located on the geometric center of wheels, then the friction torque $T = 4\frac{\sqrt{S_x^2+S_y^2}}{2}(\frac{Mg}{4}\mu_{tr}) = \frac{\mu_{tr} Mg}{2}\sqrt{S_x^{\,2}+S_y^{\,2}}$. Additionally, as the moment of inertia (I) can be express by the width (L_x) and length (L_y) of the vehicle, i.e.,

, the angular deceleration (α in Eq.(5)) around the mass center can be derived to be

$$\alpha = \frac{6\mu_{tr} g \sqrt{S_x^2 + S_y^2}}{(L_x^2 + L_y^2)} . \quad (6)$$

Thus, the angular displacement Φ_{post} will be

$$. \quad (7)$$

2-2 Collision Phase

Dynamic force analysis in the collision phase retains more complications than other two phases (Brach, 1991). Fig.1. illustrates the impact configuration of this phase.

2-2-1 Conservative of Linear Momentum

Conventionally constructing impact process applies the conservative of momentum. The linear momentum in the X- and Y-direction respectively for pre- and post-collision phases reserves following relationship (Hibbeler, 1986).

$$M_1 v_1 \cos\theta_1 + M_2 v_2 \cos\theta_2 = M_1 v_1' \cos\phi_1 + M_2 v_2' \cos\phi_2 , \quad (8)$$

$$M_1 v_1 \sin\theta_1 + M_2 v_2 \sin\theta_2 = M_1 v_1' \sin\phi_1 + M_2 v_2' \sin\phi_2 , \quad (9)$$

where the sub-scores *1* and 2 denote vehicle number; M the mass of vehicle; *V* the speed of vehicle at the instant of pre-collision; *V* the speed of vehicle at the instant of post-collision; the angle between the direction of vehicle speed and the X-axis at the instant of pre-collision; the angle between the direction of vehicle speed and the X-axis at the instant of post-collision.

2-2-2 Conservative of Angular Momentum during Collision Phase

In this part, the angular momentum around the z-axis for pre- and post-collision phases is considered. As two vehicles collide eccentrically in general, this makes them rotate after collision. Considering the conservative of angular momentum, the following equation exists, i.e., (Brach, 1991)

$$,(10)$$

where the sub-scores 1 and 2 denote vehicle number like Eq.(9); I ($= \frac{M(L_x^2 + L_y^2)}{12}$) the moment of inertia; ω the angular velocity at the instant of pre-collision; the angular velocity at the instant of post-collision; *d* the distance of the collision point to the mass center; is the angle between the X-axis and the collision point to the mass center of vehicle 1; and the angle between the Y-axis and the collision point to the mass center of vehicle 2.

2-2-3 Restoration Coefficients

The restoration coefficient during collision can be defined as (Brach, 1991)

$$\varepsilon = -\frac{(\vec{v}_a') - (\vec{v}_b')}{(\vec{v}_a) - (\vec{v}_b)} \quad (11)$$

where a and b are collision points on vehicle 1 and 2, respectively; $\vec{v}_a = \vec{v}_1$; $\vec{v}_a' = \vec{v}_1' + \vec{\Omega}_1 \times \vec{d}_1$; $\vec{v}_b = \vec{v}_2$ and $\vec{v}_b' = \vec{v}_2' + \vec{\Omega}_2 \times \vec{d}_2$. With respect to normal direction of the collision surface, Equation (11) can be rewritten as

[illegible]

$$= -\varepsilon[(v_1 \sin\theta_1 - \omega_1 d_1 \cos\alpha - v_2 \sin\theta_2 - \omega_2 d_2 \cos\beta)\sin\Gamma + (v_1 \cos\theta_1 + \omega_1 d_1 \sin\alpha - v_2 \cos\theta_2 + \omega_2 d_2 \sin\beta)\cos\Gamma] \quad (12)$$

where Γ denotes the angle between the collision surface and the Y-axis.

2-2-4 Consideration of Friction on Colliding Surfaces

As two vehicles collide, friction force originated on the collision surfaces can yield tangential impulse, which results in the variation of momentum. If the friction coefficient on the collision surfaces is μ , then the tangential impulse, , relates the normal impulse, P_n , as $P_t = \mu P_n$. According to Brach's study (1991), as sliding exists between two collision surfaces during collision, a momentum equation

$$m_1(v_1' \sin\phi_1 - v_1 \sin\theta_1)(\cos\Gamma - \mu\sin\Gamma) + m_2(v_2' \cos\phi_2 - v_2 \cos\theta_2)(\sin\Gamma + \mu\cos\Gamma) = 0 \quad (13)$$

can be addressed.

2-2-5 Consideration of Angular Momentum

The angular velocity (or angular momentum) can be influenced by angular impulse originated on collision surfaces. Brach (1998) proposed a corrected equation, which addresses the influence of angular impulse on the angular velocity of post-collision.

$$
\begin{aligned}
(\Omega_2-\Omega_1)(1+e_m)=-e_m[(\Omega_1-\omega_1)&-m_1d_1\sin\alpha(v_1'\cos\phi_1-v_1\cos\theta_1)/\mathrm{I}_1\\
&+m_1d_1\cos\alpha(v_1'\sin\phi_1-v_1\sin\theta_1)/\mathrm{I}_1\\
&-(\Omega_2-\omega_2)-m_2d_2\sin\beta(v_2'\cos\phi_2-v_2\cos\theta_2)/\mathrm{I}_2\\
&+m_2d_2\cos\beta(v_2'\sin\phi_2-v_2\sin\theta_2)/\mathrm{I}_2
\end{aligned}
\quad , (14)
$$

where e_m denotes the angular-impulse coefficient. As $e_m = -1$, it means that the angular impulse is null or neglected. Additionally, for $0 \leqq e_m \leqq 1$, $e_m = 0$ characterizes that no elastic restoration exists after collision as well as the angular velocities of two vehicles are equal, i. e., $\Omega_1 = \Omega_2$;and = 1 characterizes a completely elastic impact. In the case, the angular-impulse coefficient (e_m) is equivalent to the restoration coefficient (e).

2-2-6 Kinetic Energy Loss

Total kinetic energy of colliding vehicles looses due to friction on wheels, and the deformation of vehicle bodies resulting from collision. The kinetic energy loss, EL, can be evaluated by linear and angular velocities at instants of pre- and post-collision, i.e., (Hibbeler, 1986)

$$
\begin{aligned}
EL=&\frac{1}{2}m_1({v_{1x}}^2-{v'_{1x}}^2)+\frac{1}{2}m_1({v_{1y}}^2-{v'_{1y}}^2)+\frac{1}{2}m_1{r_1}^2({\omega_1}^2-{\Omega_1}^2)\\
&+\frac{1}{2}m_2({v_{2x}}^2-{v'_{2x}}^2)+\frac{1}{2}m_2({v_{2y}}^2-{v'_{2y}}^2)+\frac{1}{2}m_2{r_2}^2({\omega_2}^2-{\Omega_2}^2)
\end{aligned}
\quad . (15)
$$

3. Numerical Implementation and Analysis Evaluation

In the study, vehicle-body deformation due to collision was not considered yet. It was assumed that vehicle bodies remain like rectangular block, and the brake is locked so that only sliding friction is applied on tires.

3-1 Direct Derivation

Some parameters have to be collected in advance as the direct-derivation approach is applied to estimate impact configuration. They include vehicle weight, moment of inertia (vehicle length and width), distance of colliding point to mass center, angle of colliding point to mass center (with X-axis), length of braking trace before collision, speed (and direction) at braking, angle between colliding surface and Y-axis, friction coefficient between road and tire, restoration coefficient, and friction coefficient of colliding surfaces etc. Fig.2. and 3 present an illustration of parameter-input panels. For these parameters, some are specification of vehicles, such as vehicle weight, and moment of inertia etc. The others need to be identified from the accident spot. It is noted that the model in the study assumed two vehicles crash without rotation before collision.

3-2 Inverse Derivation

On the other hand, as the inverse-derivation approach is applied to estimate driving speed and reconstruct accident, some parameters are acquired for computation. They include vehicle weight, moment of inertia (vehicle length and width), distance of colliding point to mass center, angle of colliding point to mass center (with X-axis), length of braking trace before collision, sliding distance after collision, rotating angle after collision (obtained from tire trace), speed direction after collision, angle between colliding plane and Y-axis, friction coefficient between road and tire, restoration coefficient and friction coefficient of colliding surfaces etc. Fig.4. and 5 illustrate the parameter-input panels. Like the direct-derivation approach, some parameters are the specification of vehicles, and the others are identified from the accident spot. It is noted that this approach assumed (1) rotating angles can be obtained from tire traces; (2) brakes are locked after collision, then sliding friction is exerted on tires.

3-3 Analysis Evaluations

The interactive program of impact-configuration reconstruction integrated both direct-derivative and inverse-derivative modules using *NI® LabVIEW* programming language. As an example, Fig.6. and 7 illustrate partial codes for the direct- and inverse-derivation impact models, respectively. Thus, the data identified from an accident spot associated with the parameters of vehicle specification can yield driving speed through using the inverse-derivative module. Besides, a simulation of collision configuration can be processed using the direct-derivative module, as driving data of pre-collision are input.

The following demonstrates accident reconstruction results with two cases for the direct-derivation and inverse-derivation approaches each.

* Case 1 (the direct-derivation module): Vehicle 1 drives with a speed of 50 km/h from right to left. Vehicle 2 is stationary. Table 1 gives other data addressing collision configuration before impact. Fig.8. illustrates a schematic of these vehicles. By using the direct-derivation module, after collision the speed of vehicle 1 is 13.2 *m/s* along the direction of an angle 158.13° with the X-axis, and slides 12.69 m and rotates 2.77° ccw before stop. On the other hand, vehicle 2 slides with a speed of 4.36 m/s along the direction of an angle 251.57° with the X-axis, and moves 1.38 m and rotates 6.87° before stop. From the result, the kinetic energy loss is -5,637.6 *J*, which takes only *4.43%* of total initial kinetic energy (i.e., *127,315 J*), and can be accounted the yielding of a simplified model. Theoretically, the energy loss should be positive as the crush energy absorbed by vehicle body during crash is counted in the analysis model. Table 2 summarizes the analysis result. Fig.9. demonstrates the variation of different parameters such as speed, distance and rotation after collision.
* Case 2 (the inverse-derivation module): The data yielded by case 1 are utilized as the input of the inverse-derivation module, as Table 3 lists. Speed configuration before collision can be evaluated. Vehicle 1 drives with a speed of 49.98*km/h* and an angle of 179.9° with the X-axis, and vehicle 2 with a speed of 0.06 *km/h* and an orientation of 252.7°.

4. Conclusions

This study aims at developing an interactive man-machine interface using LabVIEW programming language on speed evaluation and impact configuration reconstruction of vehicles. The direct-derivation and inverse-derivation approaches were realized. Some assumptions were adopted to simplify the models. Two vehicles were considered as rectangular blocks in the whole procedure. Incompletely elastic collision was modeled, i.e., $0 \leqq \varepsilon \leqq 1$. Only sliding friction exerted on tires as brakes were locked. Deformation of vehicle bodies, which decreased total kinetic energy, was not taken into account. Thus, under-estimated driving speed can be expected. Some more complicated situations such as sliding associated with rolling friction on tires, and rollover of vehicles etc. were neglected. Although simplified conditions were assumed, this preliminary study has constructed a complete procedure for three phases of a vehicle accident. In the future, practical considerations and subsequent derivations can be further implemented to make this analysis tool more powerful.

Acknowledgement

This work was partly supported by the National Science Council of Republic of China under grant NSC89-2815-C-015-005R-E. This funding is gratefully acknowledged.

References

Brach RM (1991) *Mechanical Impact Dynamics.* John Wiley & Sons, N.Y.

Brach RM (1998) Crush Energy and Planar Impact Mechanics for Accident Reconstruction. *SAE (Society of Automotive Engineers) Paper No. 980025.*

Day TD, Hargens RL (1987) An overview of the EDCRASH computers delta-V. *SAE Paper No. 870045.*

Hibbeler RC (1986) Engineering *Mechanics- Dynamics.* Macmillan Publishing Co.

Ishikawa H, (1993) Impact Model for Accident Reconstruction- Normal and Tangential Restitution Coefficients. *SAE Paper No. 930654.*

Ishikawa H (1985) Computer simulation of automobile collision - reconstruction of accidents. *SAE Paper No. 851729.*

Manine M, Werner S (1998) Delta-V Analysis from Crash Test Data for Vehicles with Post-Impact Yaw Motion. *SAE Paper No. 980219.*

McHenry RR, McHenry BG (1997) Effects of restitution in the application of crash coefficients. *SAE Paper No. 970960.*

Prasad AK (1991) Missing vehicle algorithm (OLDMISS) reformulation. *SAE Paper No. 910121.*

Wang YW (1999) Automatically Tire Marks Identification at Accident Scene. *Journal of the Eastern Asia for Transportation Studies, Vol.3(1).*

Wirley RG, Engelmann BE (1993) Automatic contact in DYNA3D for vehicle crashworthiness analysis. *Proceedings of the ASME Winter Meeting, Symposium on Crashworthiness and Occupant Protection.*

Table 1 A list of case-1 data used in the direct-derivation approach.

<table>
<tr><td colspan="4"></td><td colspan="4">Vehicle 1</td><td colspan="4">Vehicle 2</td></tr>
<tr><td colspan="4">Weight</td><td colspan="4">1320 kg</td><td colspan="4">1570 kg</td></tr>
<tr><td>Vehicle Length</td><td>Front-to-back wheel span</td><td rowspan="2">or</td><td rowspan="2">Moment of inertia</td><td>4.47 m</td><td>3.0 m</td><td rowspan="2">or</td><td rowspan="2"></td><td>4.79 m</td><td>3.2 m</td><td rowspan="2">or</td><td rowspan="2"></td></tr>
<tr><td>Vehicle Width</td><td>Left-to-right wheel span</td><td>1.73 m</td><td>1.55 m</td><td>1.79 m</td><td>1.6 m</td></tr>
<tr><td colspan="4">Distance of colliding point to mass center</td><td colspan="4">2.39 m</td><td colspan="4">2.55 m</td></tr>
<tr><td colspan="4">Angle of colliding point to mass center (with X-axis)</td><td colspan="4">201.15°</td><td colspan="4">71.75°</td></tr>
<tr><td colspan="4">Length of braking trace before collision</td><td colspan="4">0 m</td><td colspan="4">0 m</td></tr>
<tr><td colspan="4">Speed at braking</td><td colspan="4">50 km/h</td><td colspan="4">0 km/h</td></tr>
<tr><td colspan="4">Speed direction at braking</td><td colspan="4">180°</td><td colspan="4">90°</td></tr>
<tr><td colspan="4">Angle between colliding surface and Y-axis</td><td colspan="8">45°</td></tr>
<tr><td colspan="4">Friction coefficient between road and tire</td><td colspan="8">0.7</td></tr>
<tr><td colspan="4">Restoration coefficient</td><td colspan="8">0.08</td></tr>
<tr><td colspan="4">Friction coefficient of colliding surfaces</td><td colspan="8">0.5</td></tr>
</table>

Table 2 A list of analysis result for case 1 (the direct-derivation approach).

	Vehicle 1	Vehicle 2
Vehicle speed after collision	*13.2 m/s*	*4.36 m/s*
Speed direction with X-axis	*158.13°*	*251.57°*
Rotating angle in final	*2.77°*	*6.87°*
Sliding distance in final	*12.69 m*	*1.38 m*
Kinetic energy loss	*-5637.6 J*	

Table 3 A list of case-2 data used in the inverse-derivation approach.

<table>
<tr><td colspan="4"></td><td colspan="4">Vehicle 1</td><td colspan="4">Vehicle 2</td></tr>
<tr><td colspan="4">Weight</td><td colspan="4">1320 kg</td><td colspan="4">1570 kg</td></tr>
<tr><td>Vehicle Length</td><td>Front-to-back wheel span</td><td rowspan="2">or</td><td rowspan="2">Moment of inertia</td><td>4.47 m</td><td>3.0 m</td><td rowspan="2">or</td><td rowspan="2"></td><td>4.79 m</td><td>3.2 m</td><td rowspan="2">or</td><td rowspan="2"></td></tr>
<tr><td>Vehicle Width</td><td>Left-to-right wheel span</td><td>1.73 m</td><td>1.55 m</td><td>1.79 m</td><td>1.6 m</td></tr>
<tr><td colspan="4">Distance of colliding point to mass center</td><td colspan="4">2.39 m</td><td colspan="4">2.55 m</td></tr>
<tr><td colspan="4">Angle of colliding point to mass center (with X-axis)</td><td colspan="4">201.15°</td><td colspan="4">71.75°</td></tr>
<tr><td colspan="4">Length of braking trace before collision</td><td colspan="4">0 m</td><td colspan="4">0 m</td></tr>
<tr><td colspan="4">Sliding distance after collision</td><td colspan="4">12.69 m</td><td colspan="4">1.38 m</td></tr>
<tr><td colspan="4">Rotating angle after collision (obtained from tire trace)</td><td colspan="4">2.77°</td><td colspan="4">6.87°</td></tr>
<tr><td colspan="4">Speed direction after collision</td><td colspan="4">158.13°</td><td colspan="4">251.57°</td></tr>
<tr><td colspan="4">Angle between colliding plane and Y-axis</td><td colspan="8">45°</td></tr>
<tr><td colspan="4">Friction coefficient between road and tire</td><td colspan="8">0.7</td></tr>
<tr><td colspan="4">Restoration coefficient</td><td colspan="8">0.08</td></tr>
<tr><td colspan="4">Friction coefficient of colliding surfaces</td><td colspan="8">0.5</td></tr>
</table>

Table 4 A list of analysis result for case 2 (the inverse-derivation approach).

	Vehicle 1	Vehicle 2
Vehicle speed before collision	*49.98 km/h*	*0.06 km/h*
Speed direction before collision	*179.90°*	*252.70°*
Vehicle speed at braking	*49.98 km/h*	*0.06 km/h*

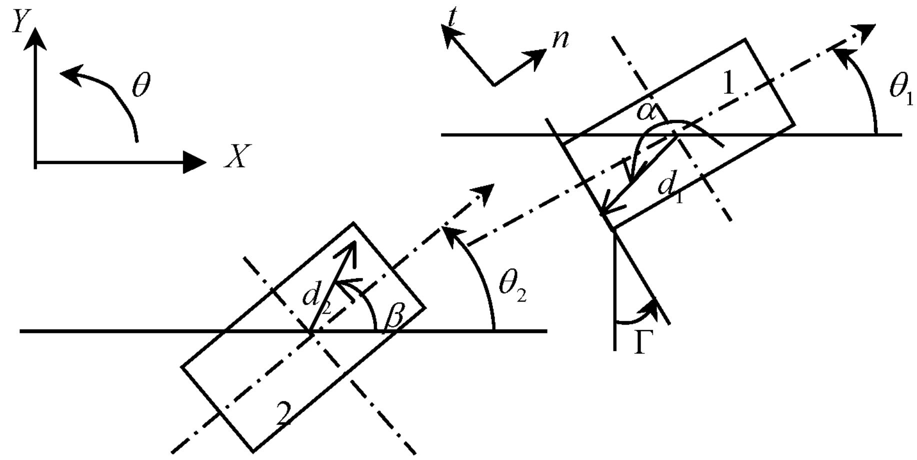

Fig.1 The free body diagram of two vehicles in a collision phase.

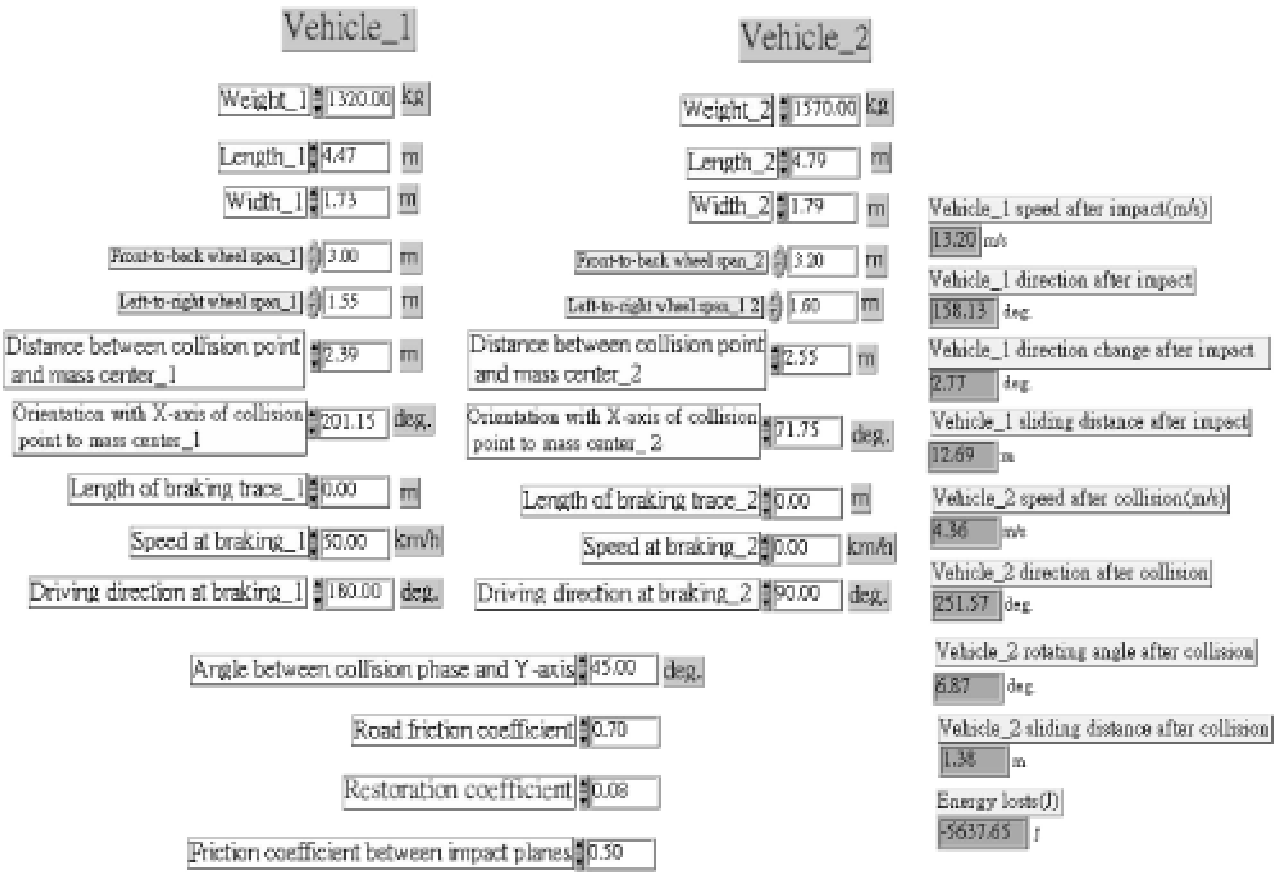

Fig.2 Human/machine interface for direct derivation (vehicle length, width, and spans of wheels are needed).

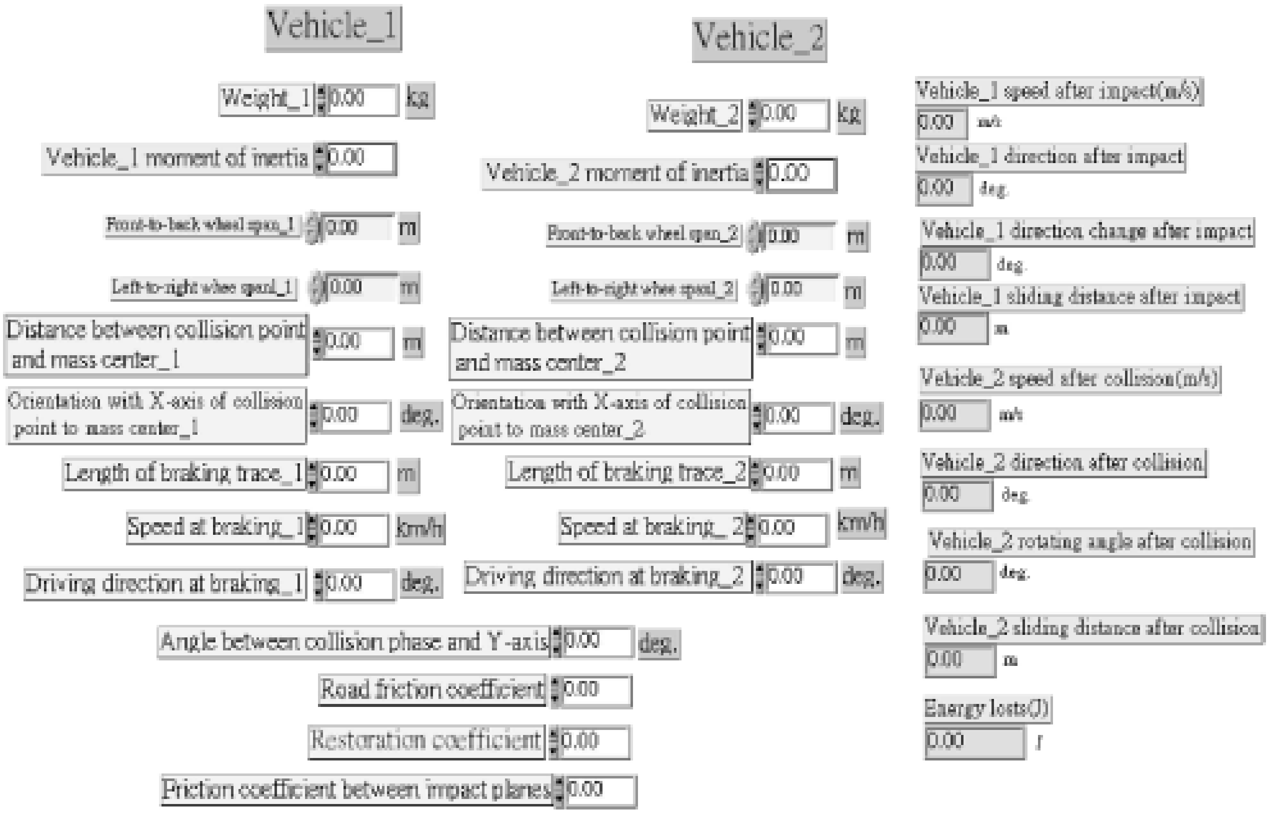

Fig.3 Human/machine interface for direct derivation (vehicle moment-of-inertias are needed).

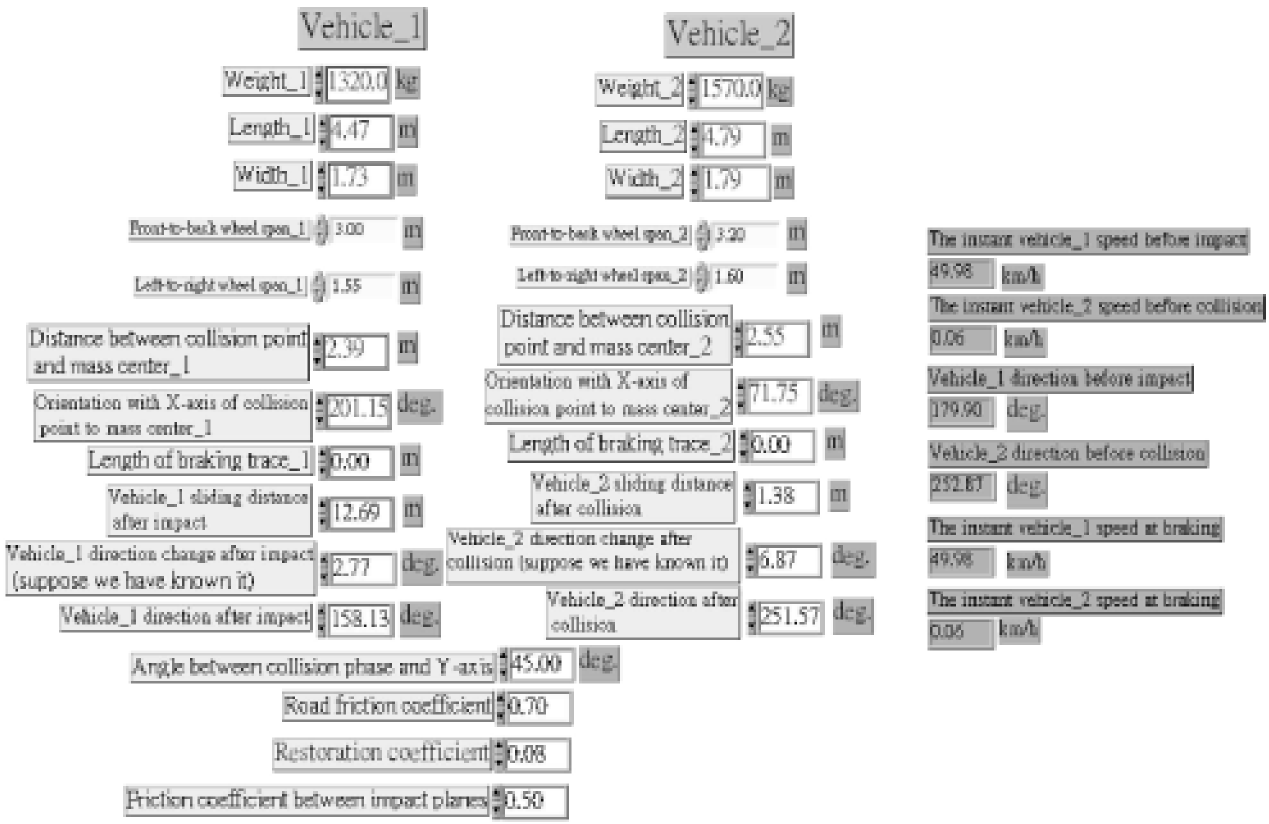

Fig.4 Human/machine interface for inverse derivation (vehicle length, width, and spans of wheels are needed).

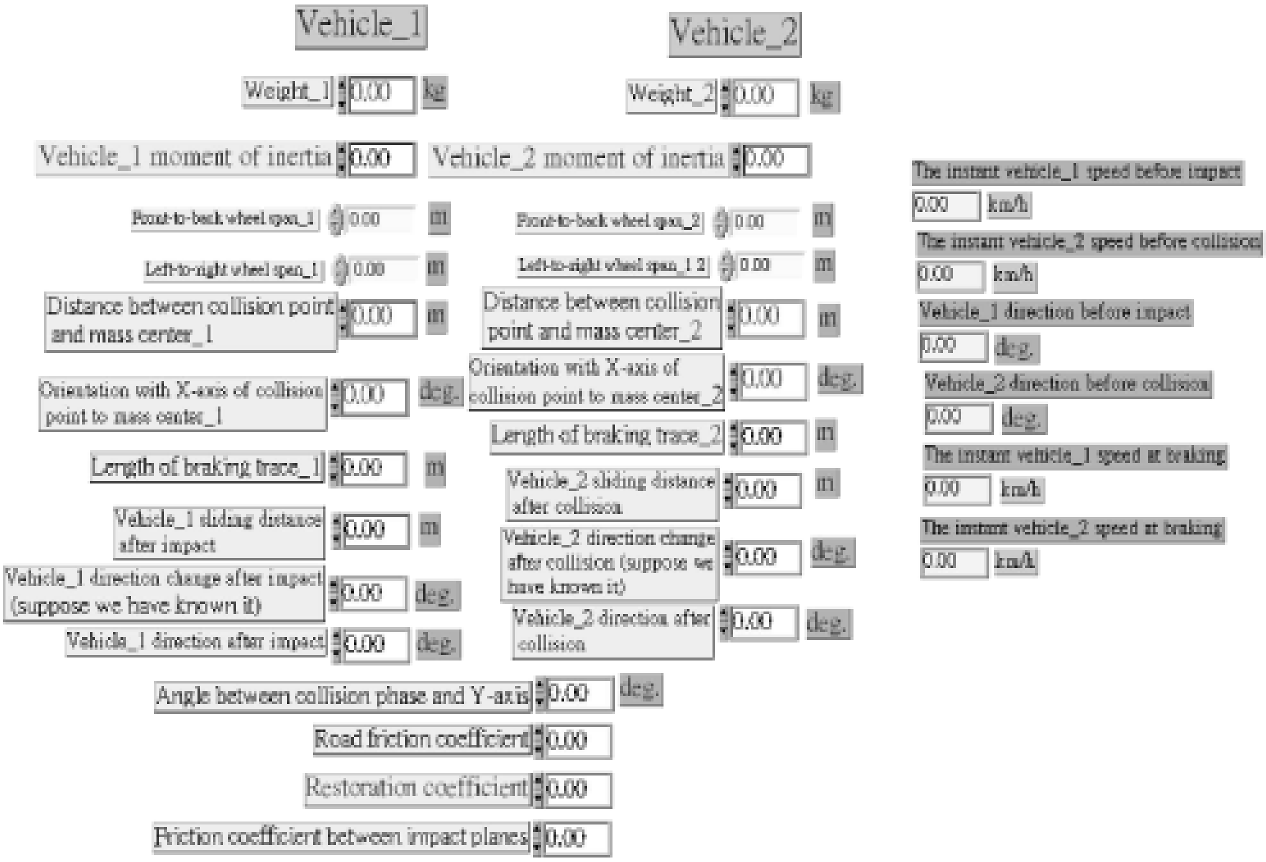

Fig.5 Human/machine interface for inverse derivation (vehicle moment-of-inertias are needed).

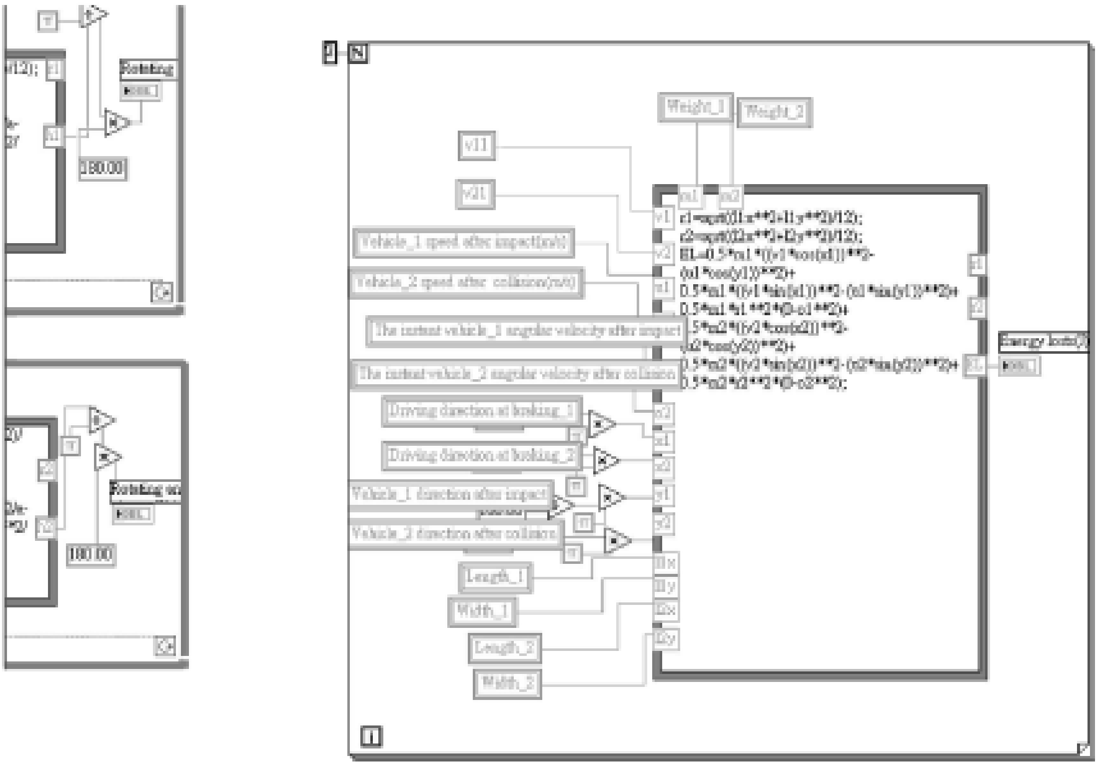

Fig.6 Partial code of direct-derivation impact model implemented in the LabVIEW programming language.

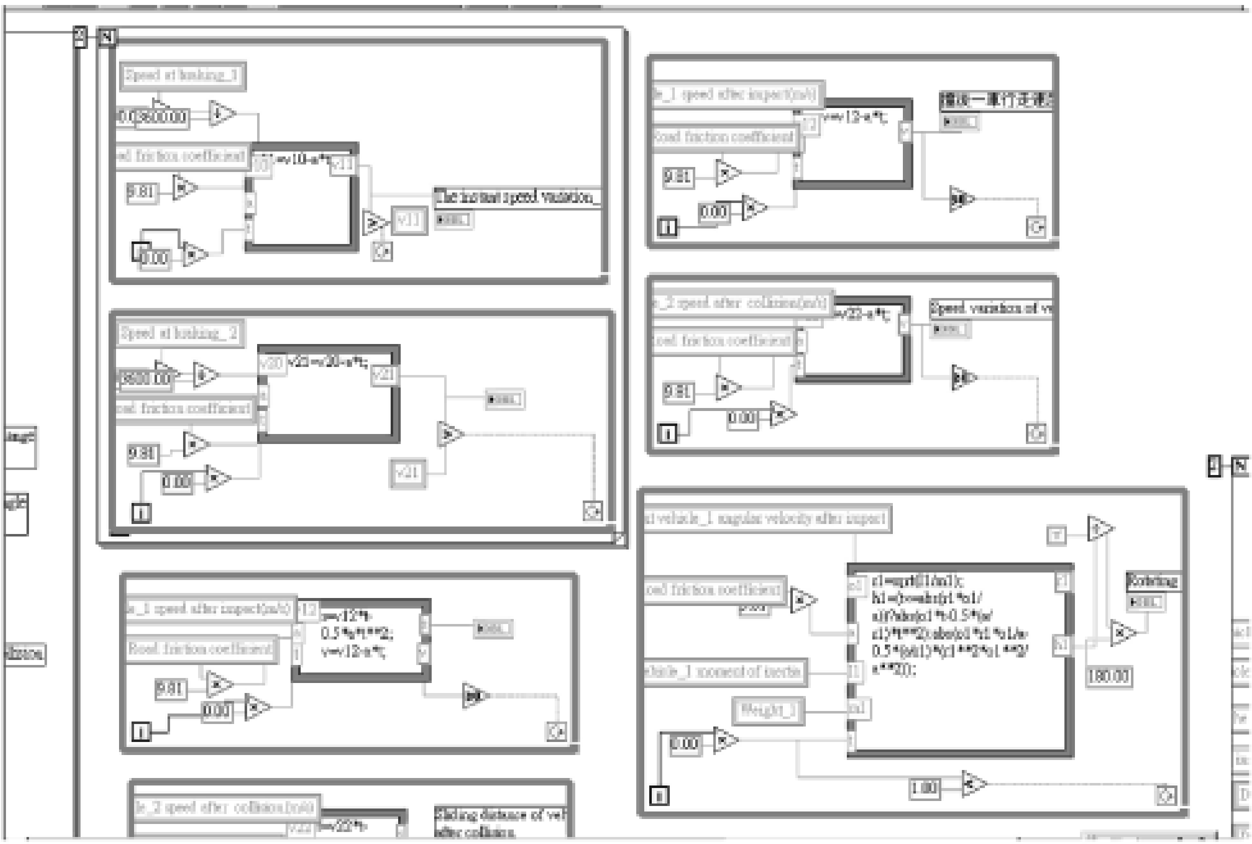

Fig.7 Partial code of inverse-derivation impact model implemented in the LabVIEW programming language.

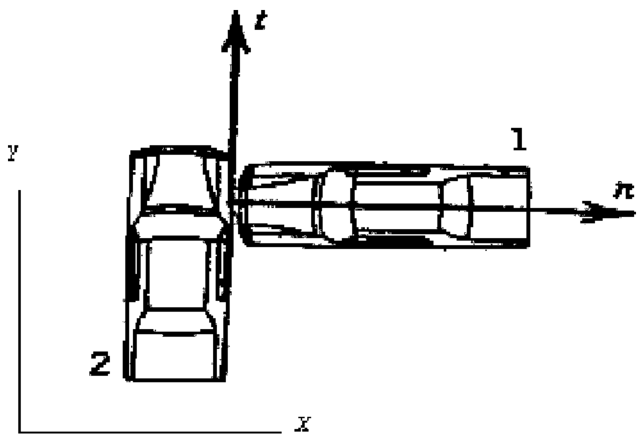

Fig.8 A schematic of case-1 collision configuration before impact.

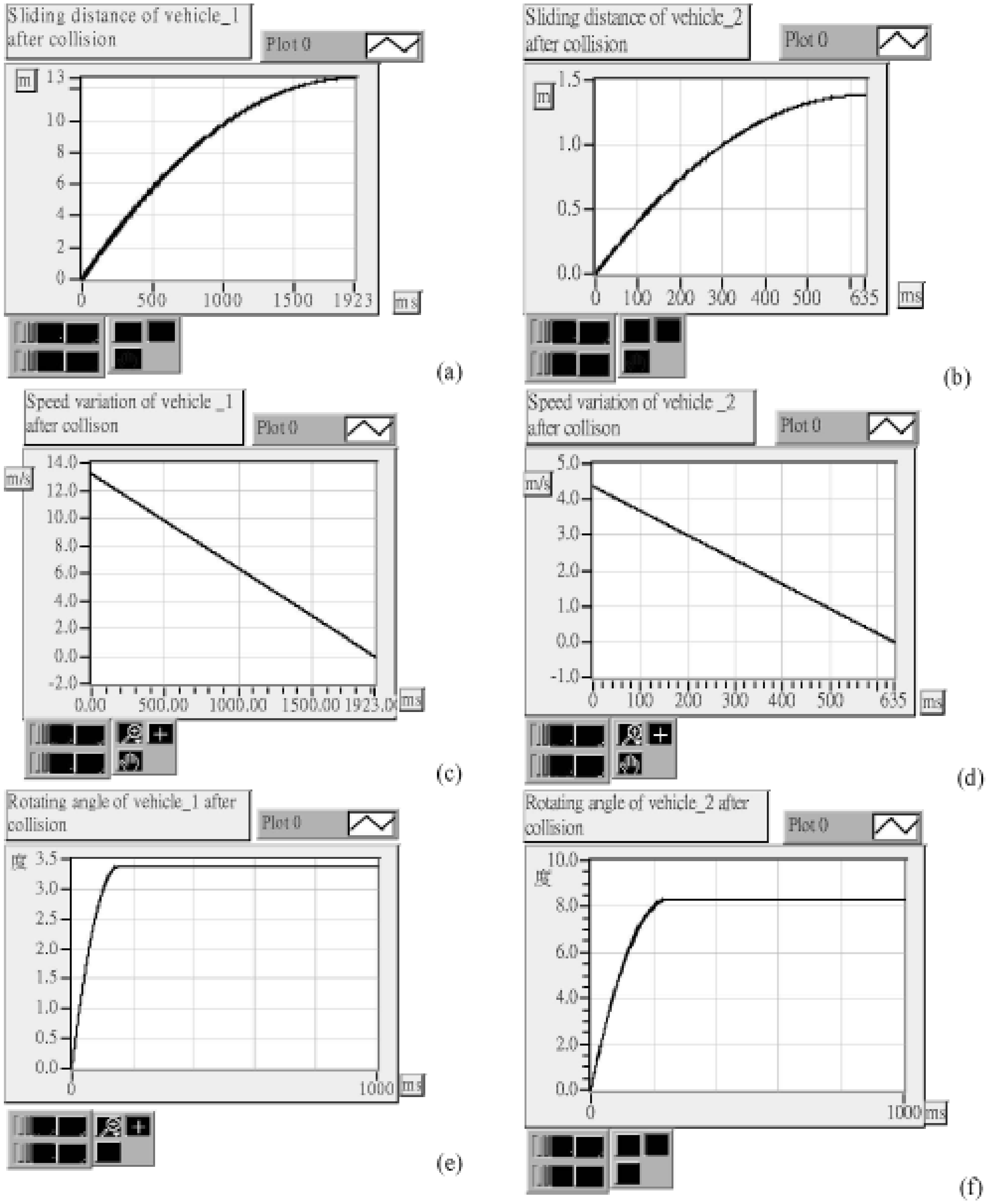

Fig.9 Chart illustration of collision configuration using the direct-derivation module. (a) and (b) Sliding distance of vehicle 1 and 2 after collision; (c) and (d) speed variation of vehicle 1 and 2 after collision; (e) and (f) rotating angle of vehicle 1 and 2 after collision.